School of Isolated and Distance Education
MATHEMATICS SPECIALIST Year 11

Test 2 2023

Section 1: Calculator Free

Time allowed for this section

Working time: 20 minutes

Mark allocation: 30 marks

PERMISSIBLE ITEMS

Standard Items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special Items: none

STANDARD FORMULAE SHEET IS PROVIDED
NO OTHER ITEMS MAY BE TAKEN INTO THE EXAMINATION ROOM

INSTRUCTIONS FOR CANDIDATES

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

All work must be done in the space provided. Should you need extra working area you may use the blank pages at the end.

Student's name: _____ Solutions _____

SIDE Teacher's name: _____

Supervisor's name: _____

Supervisor's signature:_____ Date:_____

QUESTION 1 [2, 2, 2 = 6 marks] $P \longrightarrow Q$

A true statement is "if a shape is a quadrilateral, then it is a polygon".

(a) Write the contrapositive of the statement and explain whether the contrapositive is also true.

If a shape is not a polygon $(\sim Q)$ then it is not a quadrilateral $(\sim P)$ (True)

(b) Write the inverse of the statement and explain whether the inverse is also true.

If a shape is not a quadrilateral $(\sim P)$ then it is not a polygon $(\sim Q)$ — (False)

(c) Write the <u>converse</u> of the statement and explain whether the converse is also true.

If a shape is a polygon then it is ~~not~~ a quadrilateral. — (False)

$Q \longrightarrow P$

QUESTION 2 [4 marks]

A sentence that is true or false is a statement. A premise is a statement from which a conclusion can be made.

Use the logical statements below to answer the questions:

1. Eleanor: Old men are grumpy
2. Rudolph: I'm an old man
3. Eleanor: Then you must be grumpy
4. Rudolph: ouch!
5. Eleanor: Have you been bitten by a mosquito?
6. Rudolph: Mosquitoes bite on a hot day
7. Eleanor: I think you were bitten because you have fair skin

(a) Which of the statements are conclusions?

part 3 and part 7   For both, no half mark.

(b) Which of the sentences are not statements?

part 4 and 5

(c) Which of the statements are premises?

part 2 and 6

(d) How many statements does Eleanor make?

3 statements

QUESTION 3 [6 marks]

The diameter below shows a triangle with vertices P, Q and R lie on a circle with centre O. Chord PR passes through 0. Prove by contradiction, that angle is acute $\angle QPR$ angle.

RTP: $\angle QPR$ is $< 90°$ ✓

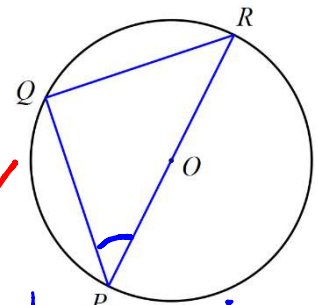Prove by contradiction: Assume that, $\angle QPR$ is $< 90°$ ✓

$\angle PQR = 90°$ (angle in semi circle) ✓

$\angle QRP + \angle QPR = 90$ (angle sum in $\triangle$) ✓

$\angle QPR = 90 - \angle QRP$ ✓

$\therefore \angle QPR < 90°$ contradict the assumption above ✓

1 For RTP

1 Assumption

3 Reasons, 1 conclusion.

QUESTION 4 [6 marks]

Diameter AB of a circle with centre O is extended to C and from C a line is drawn tangent to the circle at P. The line PT is drawn perpendicular to AB at T. Prove that

$$CA \times CB - TA \times TB = CT^2.$$

✓ $PT \cdot QT = AT \cdot BT$ (intersecting chords)

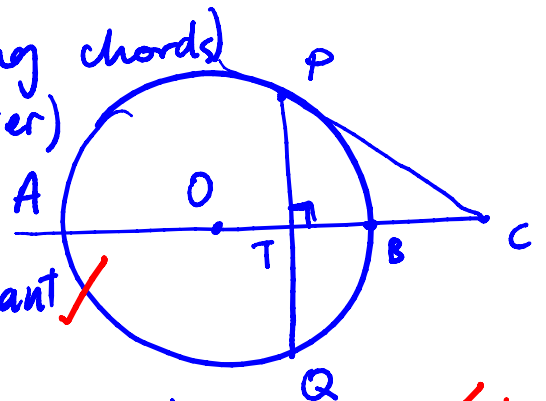$QT = PT$ (AB is a diameter)

✓ $PT^2 = AT \cdot BT$

eq. ① $- CP^2 = CA \cdot CB$ (tangent/secant theorem) ✓

eq. ② $- CP^2 = TP^2 + TC^2$ (Pythagorean's theorem) ✓ Need reason for 1 mark

eq ① = eq ②

✓ $\therefore CA \cdot CB = CT^2 + AT \cdot BT$

✓ $\therefore CA \cdot CB - TA \cdot TB = CT^2$ as required

QUESTION 5   **[2, 2, 2, 2 = 8 marks]**

(a) Write the <u>inverse</u> of the following true statement and comment on the truth of the inverse statement. $\sim P \Rightarrow \sim Q$

"If the discriminant of the quadratic formula is zero, then the quadratic has just one real root."

If the discriminant of the quad formula is Not zero, then the quad Does Not just have one real solution. True.

(b) Write the converse of the following true statement and comment on the truth of the converse statement.

$$\text{"if } x > 3 \text{ then } x > 2.\text{"}$$

If $x > 2$ then $x > 3$
x can be $= 2.5$
False

(c) Determine the truth of the following statements, using an example or counter-example to support each answer.

   i. If $z \; \mathcal{E} \; \mathbb{R} \; and \; z^3$ is an even number then z is an even number.

    Note: $\mathbb{R}$ is the set of numbers

$z^3 = 6$ (even)   then $z = \sqrt[3]{6}$ not even ✓

False ✓

   ii. If x, y $\mathcal{E} \; \mathbb{Z}$ and x > y then $x^2 > y^2$.

    Note: $\mathbb{Z}$ is the set of integers.

$x = 2), \; y = -3$
$2 > -3$ but $2^2 \not> (-3)^2$ ✓
False ✓

✓ *for supply example(s).*

End of Test

Additional page for working out